A Numerical Algorithm for Run-curve Optimization of Trains Considering DC Feeding Circuit

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Background

- Energy-saving operation in DC railway system
  - Considering feeding characteristics and interaction among several trains is essential.
  - The system has many difficult characteristics to analyze.
  - Practical discussion of optimal train operation control has just started.
Background

- Our development of optimization techniques
  - A single train using Dynamic Programming
    - COMPRAIL 2004 by H. Ko
  - A single train with energy storage using Sequential Quadratic Programming (SQP)
    - COMPRAIL 2006 by K. Matsuda (already finished)
  - Several trains using Gradient Method
    - COMPRAIL 2006 (this presentation)

\[ \text{energy} \rightarrow \min. \]

\[
T = \text{const.} \\
\int_0^T vdt = \int_0^T \int_0^T adt dt = \text{const.}
\]
Proposal of an optimization algorithm of train speed profile for practical use considering

a. Feeding circuit

b. Several trains

c. Characteristics of a train that depend on feeding voltage

   ex: Acceleration/deceleration ability, squeezing control of regenerating current and feeding loss
Mathematical formulation

Mathematical formulation is given as an optimal control problem.

- Total energy consumption at substations $\rightarrow$ Objective functional
- Kinetic equations of trains $\rightarrow$ State equations
- Circuit equations $\rightarrow$ equality constraints
- Torque limitations (and speed limitations) $\rightarrow$ inequality constraints
Definition of control input $u$

$$-1 \leq u \leq 1$$

$$f = \begin{cases} 
uf_{\text{max}}(u > 0) \\
uf_{\text{min}}(u < 0)
\end{cases}$$
## Numerical algorithms

<table>
<thead>
<tr>
<th></th>
<th>Algorithm I</th>
<th>Algorithm II</th>
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</thead>
<tbody>
<tr>
<td>Simplicity</td>
<td>Worse</td>
<td>Better</td>
</tr>
<tr>
<td>Theoretical strictness</td>
<td>Better</td>
<td>Worse</td>
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<tr>
<td>Numerical stability</td>
<td>Worse</td>
<td>Better</td>
</tr>
<tr>
<td>Applicability to large systems</td>
<td>Worse</td>
<td>Better</td>
</tr>
</tbody>
</table>
Numerical algorithms

algorithm I

Numerical algorithms

algorithm II
Dividing the objective functional for each trains

total energy consumption

\[ J = \int_0^T \sum_{m=1}^{M} E_m(t) I_m(t) \, dt \rightarrow \min \]

\[ = \sum_{j=1}^{N} U_j \rightarrow \min \]

\[ T : \text{total operating time} \]
\[ M : \text{number of substations} \]
\[ N : \text{number of trains} \]
\[ E_m : \text{voltage of } m\text{-th substation} \]
\[ I_m : \text{current of } m\text{-th substation} \]
\[ U_j : \text{objective function of } j\text{-th train} \]

which mainly consists of \( j\text{-th train’s energy consumption and feeding loss} \)

\( J \) is devided into subsets using Kirchhoff’s Current Law for distributed algorithm.
Parameters for optimizing examples

- Method: algorithm II
- PC spec: Intel Celeron 1.4GHz, 512MB
- Number of trains: 2
- Supply voltage of substations $E_s = 1500$ [V]
- Internal resistance of substations $R_s = 0.05$ [$\Omega$]
- Line resistance $R_l = 0.04$ [m$\Omega$/m]
Train movements and position of substations
Characteristics of train

(a) acceleration/deceleration

(b) squeezing control
Optimization results (Case A)

(a) $t_S=90\,[\text{s}]$  

(b) $t_S=130\,[\text{s}]$  

(c) $t_S=220\,[\text{s}]$

Speed [m/sec]

Control input

large regenerating current
Optimization results (Case B)

65[s]       130[s]       195[s]

small regenerating current
Optimization results (Case C)

40[s]  130[s]  170[s]

much small regenerating current
Energy consumption

Phase between two trains much affects energy consumption.

<table>
<thead>
<tr>
<th>Case</th>
<th>Energy Consumption [MJ]</th>
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<tbody>
<tr>
<td>A</td>
<td>244.1</td>
</tr>
<tr>
<td>B</td>
<td>280.4</td>
</tr>
<tr>
<td>C</td>
<td>329.1</td>
</tr>
</tbody>
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Summary

- The optimal operating problem of multiple trains considering DC feeding system is formulated.
- The simplified approximated numerical algorithm is performed.
- The proposed method has enough performance from numerical examples.
  - The optimal results are obtained within 1 minute.